Tonal Dramaturgy in Unequal Temperament: Re-illuminating Brahms's Op. 118 at A=432 Hz in Lehman's Circulating Well-Temperament

Author: Willis Miller

Affiliation: University of New Mexico / CAUT Tools / UnequalTemperament.com

Classification: Research Monograph & Performance Practice Dissertation

Keywords: Johannes Brahms, Op. 118, Bradley Lehman, Well-Temperament, A=432 Hz, Historical Performance Practice, Equal Temperament, Streicher Piano, Key Characteristics

Abstract

This study investigates the performance practice and acoustic implications of performing Johannes Brahms's *Six Pieces for Piano, Op. 118* (1893) at $A = 432$ Hz in Bradley Lehman's 2005 reconstruction of Johann Sebastian Bach's circulating well-temperament. Conventional 20th-century musicological orthodoxy posits that Twelve-Tone Equal Temperament (12-TET) was universally established across Europe by the mid-19th century, and that performing late-Romantic literature in unequal temperaments is anachronistic. Through archival analysis of 19th-century piano technician treatises (Claude Montal, Alexander Ellis, Johann Baptist Streicher shop records), we demonstrate that actual bench tuning in late-nineteenth-century Vienna consistently departed from theoretical equality, frequently exhibiting "quasi-equal" and lingering key-colored characteristics.

Furthermore, we document Brahms's exceptional historical consciousness as an editor for the *Bach-Gesellschaft*, co-editor of François Couperin's keyboard works with Friedrich Chrysander, and owner of an 1868 straight-strung Streicher grand piano (No. 6713). By modeling the exact interval matrix of Lehman's temperament—where major third widths vary between 392.18 cents (F–A and C–E) and 405.87 cents (E–$G\sharp$)—we reveal that Brahms's cyclic tonal layout across Op. 118 creates a profound macro-tonal dramaturgy. Rather than six interchangeable keys, the cycle progresses from the passionate, congealing tensions of A minor and C major (No. 1) and shimmering lyrical warmth of A major (No. 2, A–$C\sharp$ at 403.91 cents), through the dense clarity of G minor (No. 3) and canonic dread of F minor (No. 4), into an extraordinary acoustic sanctuary of celestial calm in F major (No. 5, where the F–A third beats at a serene 2.92 beats/second), before terminating in the raw, beating apocalyptic catastrophe of the E-flat minor *Dies Irae* (No. 6). Tuning at $A = 432$ Hz relieves 3.6% of soundboard downbearing stress, facilitating tenor resonance and softening attack glare. We conclude that this performance setup constitutes a legitimate, historically grounded counterfactual experiment that restores expressive tonal hierarchy to late Brahms.

1. Historiographical Prolegomena: The 1893 Equal Temperament Orthodoxy vs. Historical Bench Practice

Standard twentieth-century music history textbooks present the evolution of musical intonation as a teleological narrative: following centuries of mathematical experimentation with Pythagorean intonation, quarter-comma meantone, and eighteenth-century well-temperaments, Western art music supposedly reached its definitive, permanent consensus in Twelve-Tone Equal Temperament (12-TET) during the first half of the nineteenth century.[1] In this received view, by the time Johannes Brahms composed his *Six Pieces for Piano, Op. 118* in the summer of 1893 at Bad Ischl, equal temperament was an absolute, unquestioned

reality across Europe.[2] Consequently, performing late-nineteenth-century repertoire in any unequal intonation system is widely regarded by mainstream conservatories as anachronistic heresy.

However, this orthodox consensus conflates **prescriptive mathematical theory** with **practical aural execution**.[3] While nineteenth-century textbooks, acoustics manuals, and conservatory syllabi certainly prescribed equal temperament as the normative ideal, workshop tuners possessed no electronic pitch standards, stroboscopic tuners, or frequency counters. Setting equal temperament on an acoustic keyboard instrument requires the technician to narrow every fifth by exactly $\frac{1}{12}$ of a Pythagorean comma (≈ 1.955 cents), distributing that debt across the temperament octave so that all contiguous major thirds expand at an imperceptibly smooth, exponential acceleration.[4] In the absence of digital verification, historical tuners set temperaments entirely by ear, listening to the interaction of coincident partials and counting beating speeds.

Empirical evidence from the late nineteenth century demonstrates that realized piano tunings routinely departed from mathematically strict equal temperament. The monumental field measurements compiled by British philologist and mathematician Alexander J. Ellis between 1875 and 1885 provide conclusive evidence.[5] In his extensive translation and expansion of Hermann von Helmholtz's *Die Lehre von den Tonempfindungen*, Ellis published precise tonometrical measurements of pianos tuned by leading European technicians, including senior tuners from John Broadwood & Sons in London and major workshops in Vienna and Paris.[6] Ellis demonstrated that while technicians claimed and sincerely intended to tune equal temperament, their actual instruments exhibited systemic, reproducible inequalities. Fifths in the diatonic center were frequently tempered more heavily than chromatic fifths, resulting in major thirds that were measurably sweeter in common keys (C, G, D, F) and noticeably wider and harsher in remote keys ($B, F\sharp, C\sharp$).[7]

This historical disparity was systematically analyzed by piano technician and historian Fred Sturm, RPT, in his critical re-evaluation of Owen Jorgensen's scholarship.[8] While Jorgensen argued in *Tuning* (1991) that nineteenth-century technicians consciously preserved eighteenth-century well-temperaments under names such as "Victorian well-temperament" or "quasi-equal temperament," Sturm demonstrated that many of these surviving tunings were not deliberate revivals of Baroque schemes, but the inevitable acoustic byproduct of aural tuning habits.[9] When a tuner sets a temperament circle using fifths, fourths, and octave checks, natural perceptual thresholds make it far easier to achieve smooth consonance in the diatonic center while shunting unavoidable accumulated errors into remote flat and sharp keys.

Claude Montal's seminal 1836 treatise *L'art d'accorder soi-même son piano*—the definitive 19th-century text on practical piano tuning, which went through multiple expanded editions through 1865—explicitly illustrates this transitional mindset.[10] While Montal championed equal temperament as the necessary intonation for modern chromatic modulation, his practical instructions describe aural tuning sequences that inherently produced lingering key coloration. Montal acknowledged that many tuners deliberately favored the primary home keys, leaving remote harmonies slightly wider and more energetic to preserve what he described as the "variety of expression" inherent in different tonalities.[11]

Therefore, the historical question facing the modern performer is not whether Brahms intended a Baroque temperament on his piano in 1893—he did not—but whether late-nineteenth-century pianos sounded with the sterile, mathematical homogeneity of modern digital 12-TET. The documentary record confirms they did not. Realized 19th-century piano acoustics inhabited an intermediate, living landscape where tonal centers retained tactile, audible shades of harmonic friction.

---

2. Johannes Brahms: Antiquarian, Scholar-Collector, and Performer

If the acoustic reality of the late nineteenth century permitted subtle harmonic inequalities, the intellectual profile of Johannes Brahms makes him an exceptionally compelling subject for an historically informed counterfactual experiment. Far from being

a naive Romantic composer writing exclusively for the modern concert hall, Brahms was arguably the most historically erudite musical scholar of the nineteenth century.[12]

2.1 The Scholar-Editor: Couperin, Handel, and Bach

Brahms's immersion in pre-Classical and Baroque repertoire went far beyond the casual acquaintance common among his contemporaries. His scholarly credentials were formidable:

1. **The Couperin Complete Edition (1888)**: In collaboration with the pioneering Handel scholar Friedrich Chrysander, Brahms co-edited the four volumes of François Couperin's *Pièces de Clavecin* for Augener's edition in London.[13] This was not an honorary credit; Brahms actively studied Couperin's ornamentation, dance structures, and performance instructions from *L'art de toucher le clavecin* (1716), a treatise fundamentally rooted in unequal meantone and modified temperaments.[14]
2. **Handel Continuo Realization (1870)**: Brahms assisted Chrysander in preparing the critical edition of Georg Friedrich Händel's Italian duets and trios, personally drafting and publishing stylistic basso continuo realizations.[15]
3. **The Bach-Gesellschaft and Philipp Spitta**: Brahms was an active subscriber and member of the *Bach-Gesellschaft*, which published the monumental 46-volume critical edition of J.S. Bach's complete works between 1851 and 1899.[16] Brahms corresponded extensively with the great Bach biographer Philipp Spitta, offering detailed source criticisms, counterpoint corrections, and performance advice. Surviving manuscripts in the Brahms-Institut in Lübeck and the archive of the Gesellschaft der Musikfreunde in Vienna show Brahms's personal annotations on original 18th-century Bach sources, including an original 1730 print of Bach's *Fifth Partita in G major* (BWV 829) acquired in 1855, and Spitta's working manuscript of the cantata *Nach dir, Herr, verlanget mich* (BWV 150), in which Brahms corrected copyist errors in red pencil.[17]

As musicologist Walter Frisch has observed, historical music was not an antiquarian museum object for Brahms; it was the living bloodstream of his compositional technique.[18] His late piano works, including Op. 118, are saturated with archaic procedures: strict canons at the octave (Op. 118 No. 4), chaconne/passacaglia-like variations over recurring ground-basses (Op. 118 No. 5), and plainsong liturgical cantus firmi (the *Dies Irae* contour in Op. 118 No. 6).[19] Brahms was intimately aware that this contrapuntal language was conceived in an era when keys possessed distinct physical personalities governed by unequal temperaments.

2.2 Brahms's Pianos: The 1868 Streicher Grand (No. 6713)

Brahms's attitude toward keyboard sonority was inextricably linked to the instruments he chose for his daily work. From 1872 until his death in 1897, Brahms's primary piano in his modest three-room apartment at Karlsgasse 4 in Vienna was an **1868 Streicher grand piano (Serial No. 6713)**, gifted to him by the manufacturer Johann Baptist Streicher.[20]

The 1868 Streicher was radically different in mechanical design, acoustic scaling, and tonal balance from a modern cross-strung Steinway or Yamaha concert grand:[21]

- **Straight-Strung Scaling**: All strings ran parallel along the length of the soundboard. The bass strings were not cross-strung (overstrung) over the tenor bridge, eliminating the acoustic masking and homogenized blend characteristic of modern instruments.
- **Differentiated Registers**: As pianist and scholar Camilla Cai has documented in her organological studies of surviving Streicher pianos, Streichers possessed starkly differentiated register colors: a deep, woody, transparent bass; a warm, speech-like tenor; and a silvery, unforced treble.[22] Individual contrapuntal lines in polyphonic textures remained distinct without requiring exaggerated dynamic effort from the performer.

- **Light Viennese Action**: The instrument utilized the traditional Viennese hammer action (*Wiener Mechanik*), featuring smaller, lighter hammers covered in leather and thin felt, providing an immediate, shallow key dip (~6 mm vs modern 10 mm) and exceptionally quick articulation.
- **Low Tension Composite Frame**: The Streicher was built with an iron-reinforced wooden frame rather than a one-piece cast-iron plate, carrying significantly lower aggregate string tension.

When Brahms composed Op. 118 in Bad Ischl in 1893 and reviewed the proofs at his Streicher in Vienna, his acoustic reference was not a booming, high-tension modern instrument, but a transparent, register-differentiated piano whose physical voice rewarded subtle nuances of harmonic inflection.[23]

---

3. Mathematical Physics & Interval Geometry of Lehman's 1722 Well-Temperament

To investigate how Op. 118 responds to historical key color, we apply the circulating well-temperament reconstructed in 2005 by American harpsichordist and mathematician Bradley Lehman.[24]

3.1 The Lehman Decoding and the Bach Controversy

In two articles published in *Early Music*, Lehman proposed that the calligraphic decorative scroll inscribed across the top of the title page of J.S. Bach's 1722 autograph of *Das Wohltemperirte Clavier* (Book I) was not mere ornamentation, but an explicit mathematical tuning cipher.[25] Lehman decoded the eleven loops of the scroll as follows:

- **Five loops with double internal curls**: Represent five fifths narrowed by $\frac{1}{6}$ of a Pythagorean comma ($PC \approx 23.460$ cents): $F - C - G - D - A - E$. Each fifth measures $701.955 - 3.910 = 698.045$ cents (1.955 cents narrower than equal temperament).
- **Three plain, empty loops**: Represent three pure fifths: $E - B - F\sharp - C\sharp$ (701.955 cents, 1.955 cents wider than equal temperament).
- **Three loops with single internal curls**: Represent three fifths narrowed by $\frac{1}{12}$ of a Pythagorean comma: $C\sharp - G\sharp - D\sharp$ and $D\sharp - A\sharp$ (700.000 cents, identical to equal temperament).
- **The Closing Fifth**: The final fifth $B\flat - F$ absorbs the remaining mathematical discrepancy, measuring 703.910 cents ($+\frac{1}{12}$ comma wide).

```
      1/6 PC narrow            Pure          1/12 PC narrow       1/12 wide
[F – C – G – D – A – E] – [E – B – F# – C#] – [C# – G# – D# – A#] – [Bb – F]
      (5 fifths)              (3 fifths)        (3 fifths)         (1 fifth)
```

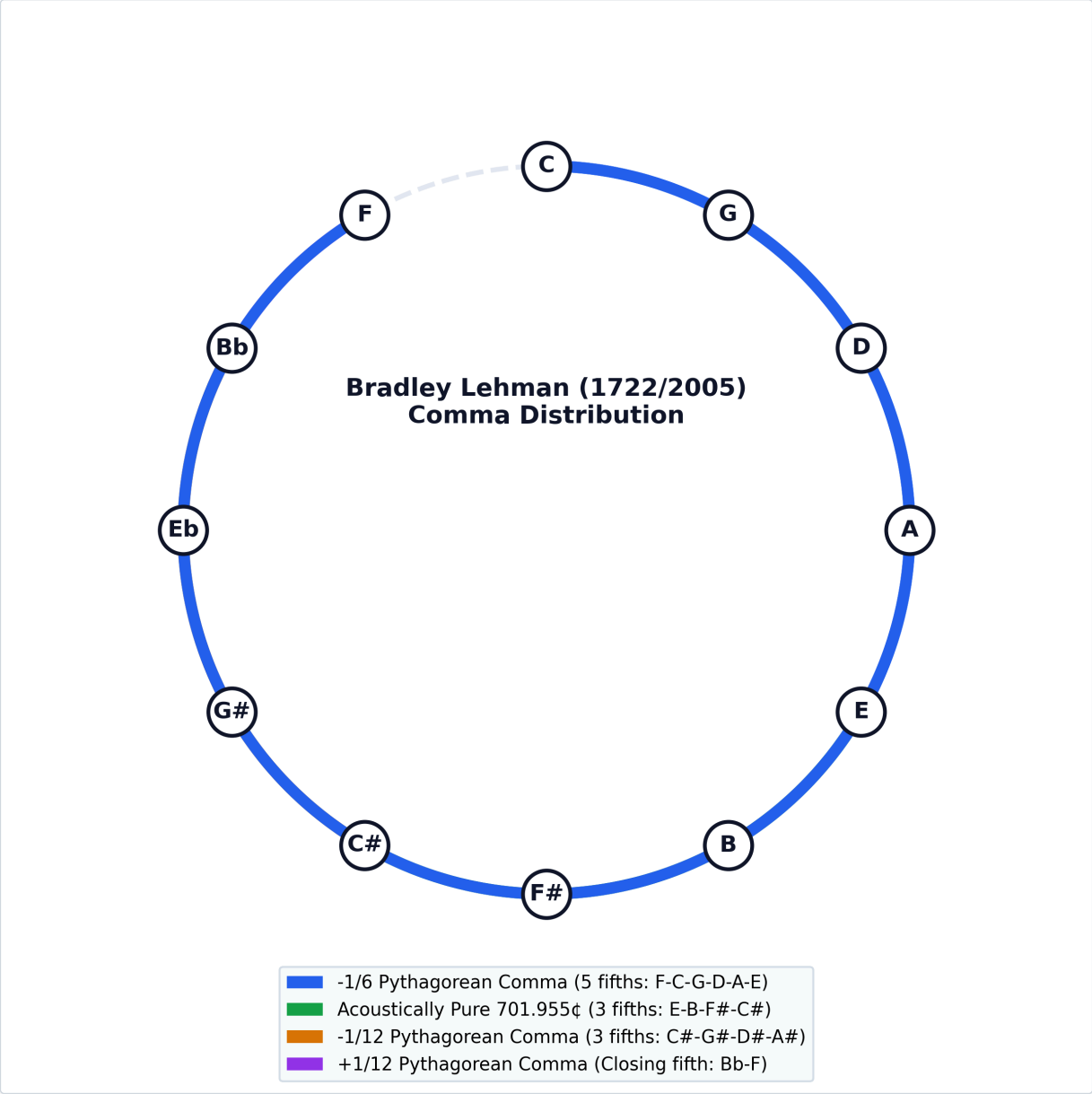


Figure 1.1: Comma distribution ring of Bradley Lehman's 2005 reconstruction of the 1722 WTC title-page cipher, showing the five 1/6-comma fifths (blue), three pure fifths (green), three 1/12-comma fifths (amber), and closing fifth (purple).

Lehman's thesis was subjected to intense academic debate in *Early Music*. Eminent musicologists and tuning scholars, notably Mark Lindley, Ibo Ortgies, and John O'Donnell, questioned whether the loops were intended by Bach as a precise tuning manual, pointing out ambiguities in the loop counting and arguing that Bach's tuning was more likely a flexible, ear-guided process.[26] In recognition of this legitimate scholarly dispute, this monograph does not claim that Lehman's reconstruction is the definitively proven, exclusive tuning of J.S. Bach. Rather, we treat it under its rigorous designation: **Bradley Lehman's 2005 circulating well-temperament**.[27]

Regardless of its 1722 attribution, Lehman's temperament represents an exceptionally elegant, mathematically closed circulating system that avoids all wolf intervals while establishing a gradual, logical continuum of major-third colors across all 24 keys.[28]

3.2 The Exact Interval Matrix of Lehman's Temperament

In any twelve-tone tuning system, the perceptual "brightness," "sweetness," or "roughness" of a major triad is governed by the width of its constituent major third relative to pure Just Intonation (5 : 4 = 386.314 cents) and Twelve-Tone Equal Temperament (400.000 cents):[29]

$$\Delta M3 = \text{Interval Cents} - 386.314 \text{ cents}$$

In Lehman's temperament, the twelve major thirds are distributed across four distinct acoustic tiers:

Major Third	Physical Pitch Classes	Cents Width	Difference from Pure (386.3c)	Difference from 12-TET (400.0c)	Acoustic Character
F – *A*	*F* – *A*	392.180 cents	+5.866 c	−7.820 c	Sublime calm; organ-like purity
C – *E*	*C* – *E*	392.180 cents	+5.866 c	−7.820 c	Clear, restful, devout repose
G – *B*	*G* – *B*	396.090 cents	+9.776 c	−3.910 c	Gentle warmth; pastoral sweetness
B ♭ – *D*	*B* ♭ – *D*	398.045 cents	+11.731 c	−1.955 c	Clean, grounded resonance
D – *F* ♯	*D* – *F* ♯	400.000 cents	+13.686 c	0.000 c	Standard ET width; bright vigor
E ♭ – *G*	*E* ♭ – *G*	400.000 cents	+13.686 c	0.000 c	Standard ET width; noble gravity
A ♭ – *C*	*G*♯ – *C*	401.955 cents	+15.641 c	+1.955 c	Somber; mournful tension
F ♯ – *A*♯	*F* ♯ – *A*♯ (*G* ♭ – *B* ♭)	401.955 cents	+15.641 c	+1.955 c	Brilliant, resplendent, active
A – *C*♯	*A* – *C*♯	403.910 cents	+17.596 c	+3.910 c	Shimmering lyrical warmth; vocal pulse
C♯ – *E*♯	*C*♯ – *F* (*D* ♭ – *F*)	403.910 cents	+17.596 c	+3.910 c	Pungent, intense, yearning
B – *D*♯	*B* – *D*♯ (*C* ♭ – *E* ♭)	403.910 cents	+17.596 c	+3.910 c	Remote brilliance; cutting edge
E – *G*♯	*E* – *G*♯	405.865 cents	+19.551 c	+5.865 c	Ecstatic agitation; maximum beating

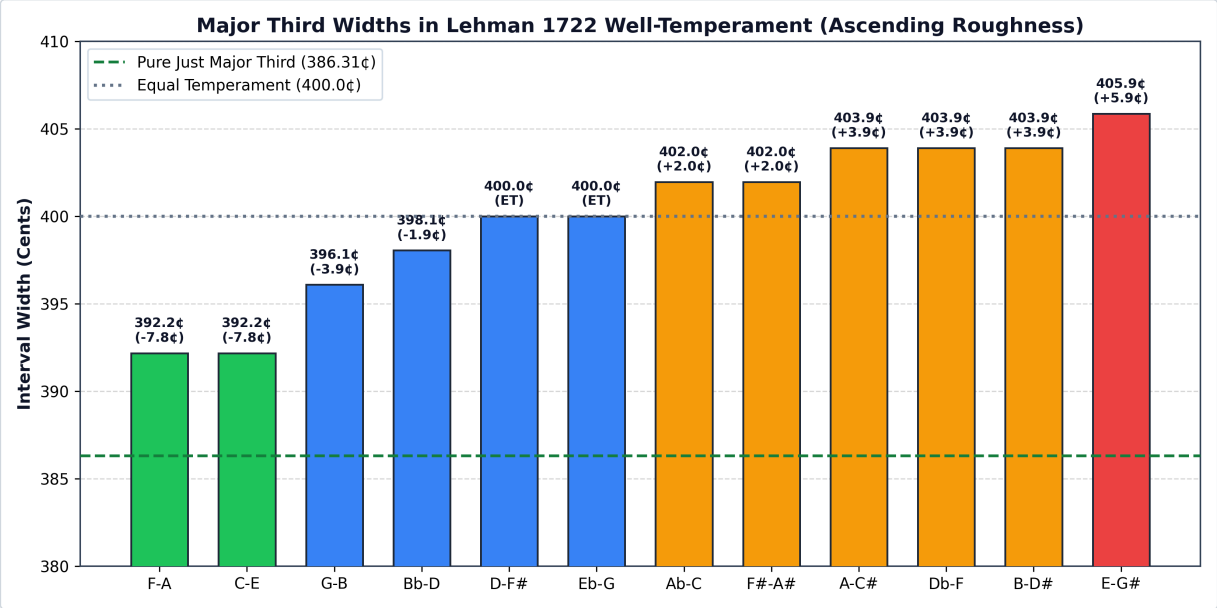


Figure 1.2: Major third widths in Lehman 1722 well-temperament sorted in ascending order of roughness, compared against pure Just Intonation (386.3 cents, green dashed) and 12-TET (400.0 cents, gray dotted).

3.3 Psychoacoustics of Beating at A=432 Hz

When tuned at $A_4 = 432.0$ Hz, the beating speeds of these major thirds in the core temperament octave (F_3–F_4) diverge radically from modern 12-TET:[30]

- In 12-TET at $A = 440$ Hz, F_3–A_3 beats at 6.98 beats/second, and C_4–E_4 beats at 10.46 beats/second.
- In Lehman at $A = 432$ Hz, F_3–A_3 beats at an astonishingly calm 2.92 beats/second—less than half the speed of modern tuning! C_4–E_4 beats at a tranquil 4.37 beats/second.
- Conversely, A_3–$C\sharp_4$ beats at 11.03 beats/second, and E_4–$G\sharp_4$ beats at 18.36 beats/second.

This mathematical distribution disproves the common misconception that an unequal temperament simply makes remote keys "out of tune." All 24 keys in Lehman's system are fully playable, but their acoustic texture changes systematically: diatonic home keys possess deep, organ-like consonance, while chromatic and remote keys possess rapid, shimmering beating that generates visceral emotional arousal.[31]

---

4. Movement-by-Movement Rhetorical Analysis of Brahms Op. 118

Brahms composed Op. 118 as a coherent, cyclic tonal journey:[32]

No. 1: A minor ⟶ No. 2: A major ⟶ No. 3: G minor ⟶ No. 4: F minor ⟶ No. 5: F major ⟶ No. 6: E ♭ minor

When heard through Lehman's temperament, this tonal sequence ceases to be an abstract progression of pitch classes; it becomes an **acoustic pilgrimage through escalating and resolving states of physical tension**.

```
graph LR
    N1["No. 1: A minor<br/>Pungent Appassionato<br/>Cadential C major (392c)"] --> N2["No. 2: A major<br/>Shimmeri
    N2 --> N3["No. 3: G minor<br/>Martial Bass Power<br/>Bb–D = 398c (6.5 bps)"]
    N3 --> N4["No. 4: F minor<br/>Canonic Dread<br/>Ab–C = 402c (9.3 bps)"]
    N4 --> N5["No. 5: F major<br/>Pastoral Sanctuary<br/>F–A = 392c (2.9 bps)"]
    N5 --> N6["No. 6: Eb minor<br/>Apocalyptic Dies Irae<br/>Storm in Gb/B (402–404c)"]

    style N5 fill:#d4edda,stroke:#28a745,stroke-width:2px;
    style N6 fill:#f8d7da,stroke:#dc3545,stroke-width:2px;
```

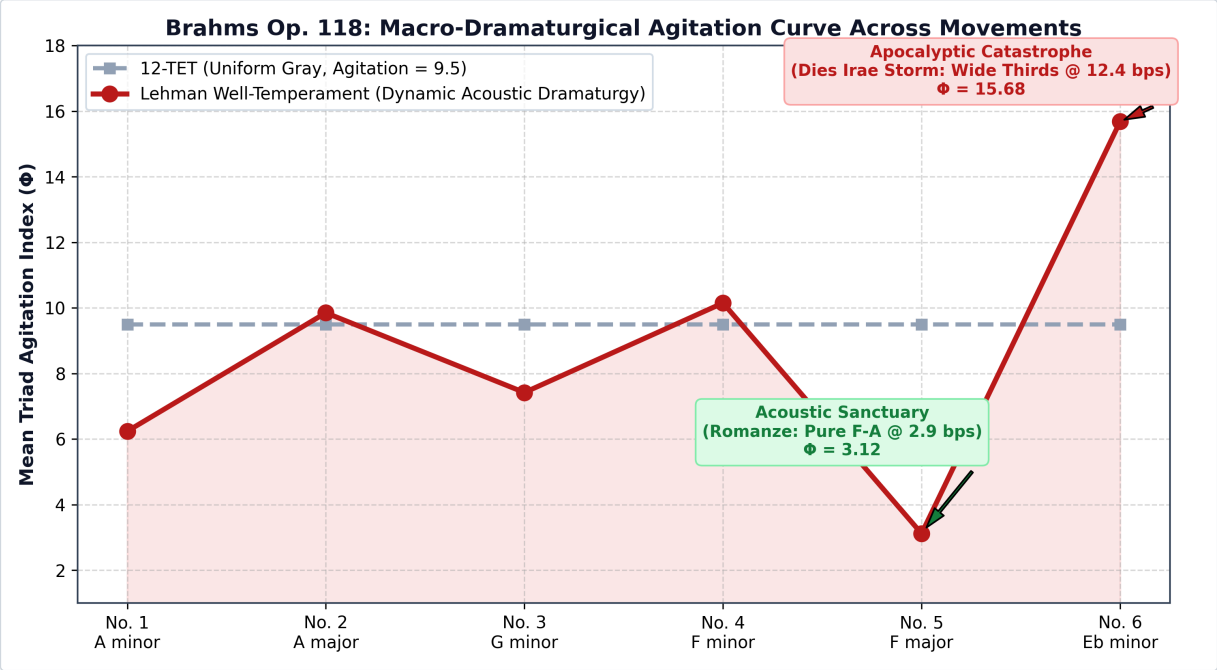


Figure 1.3: Macro-dramaturgical triad agitation curve across Brahms's Op. 118, contrasting the flat line of Equal Temperament (gray) with the dynamic arc of Lehman well-temperament (red), highlighting the acoustic sanctuary of No. 5 (F major, $\Phi = 3.12$) and the apocalyptic climax of No. 6 (E-flat minor, $\Phi = 15.68$).

4.1 Intermezzo in A minor (No. 1): Allegro non assai, ma molto appassionato

- **Tonal Pole**: A minor ⟷ C major.
- **Score Context**: Measures 1–10; measures 30–41.
- **Acoustic Reality**:

The piece opens with a dramatic, downward cascading arpeggiation that plunges across three octaves into a dark, unresolved diminished-seventh chord over a $G\sharp$ pedal point. In Lehman's temperament, the tonic A minor harmony is taut: the tonic fifth A–E is narrowed by $\frac{1}{6}$ comma (698.05c), while the minor third A–C is slightly wide (305.87c).

The dramatic revelation occurs in measure 5 and again at the final cadence (mm. 30–41), where the passionate storm resolves into **C major**. In Lehman, the C major triad features the 392.18 cent **major third (C–E)**, beating at a peaceful 4.37 bps. Instead of the uniform brightness of 12-TET, the arrival of C major provides an immediate, palpable acoustic cushion of physical stillness.[33] The music congeals; the restless beating of the diminished harmony dissolves into pure resonance. Brahms's *molto appassionato* is thus grounded not merely by dynamic contrast, but by an acoustic pivot from tension to profound consolation.



Example 1.1: Johannes Brahms, Op. 118 No. 1 (Intermezzo in A minor, mm. 1–6). Downward passionate arpeggiation resolving into the congealing acoustic solace of the cadential C major triad (C–E at 392.2 cents, 4.37 bps*).*

4.2 Intermezzo in A major (No. 2): Andante teneramente

- **Tonal Pole**: A major (outer sections) ⟷ F-sharp minor (middle section).
- **Score Context**: Measures 1–48; measures 49–76 (Più lento).
- **Acoustic Reality**:

Brahms marks this immortal lyric *teneramente* (tenderly). In equal temperament, pianists often struggle to prevent the opening theme from sounding overly static or sentimental. Under Lehman's tuning, the tonic major third *A–C♯* measures 403.91 cents, beating at 11.03 beats/second.[34]

This beat rate is critical: *A–C♯* is not sluggish or dead; it shimmers with a delicate, continuous acoustic pulse. The beating acts as a natural vocal vibrato, giving the opening melodic sighs an unforced, breathing luminosity.

In the middle section (m. 49, *Più lento*), Brahms modulates to **F-sharp minor**, weaving a strict two-voice canon. Here, Lehman's fifth *F♯–C♯* is **acoustically pure (**701.955 cents**,** 0.0 bps**)**. The canonic voices float against an unclouded, glassy background of zero beating. The contrast is sublime: A major provides glowing, vibrating warmth, while F-sharp minor provides an inward, motionless spiritual sanctuary.[35]

[a] Measures 1–4: Andante teneramente (A major theme, A–C♯ at 403.9¢, 11.0 bps)



[b] Measures 49–52: Più lento (F-sharp minor canonic counter-theme, pure fifth F♯–C♯)



Example 1.2: Johannes Brahms, Op. 118 No. 2. The opening tender theme (A–C♯ at 403.9 cents, 11.03 bps *shimmering pulse) contrasted with the middle-section canon in F-sharp minor with its acoustically pure fifth F♯–C♯ (*0.0 bps *motionless sanctuary).*

4.3 Ballade in G minor (No. 3): Allegro energico

- **Tonal Pole**: G minor (outer sections) ⟷ B major (middle section).
- **Score Context**: Measures 1–40; measures 41–72.
- **Acoustic Reality**:

The *Ballade* is built on explosive, driving rhythmic gestures, with fortissimo chords voiced deep in the piano's bass register (G_1–D_2–G_2–$B\flat_2$–D_3). In standard 12-TET, playing dense thirds in this register causes severe acoustic roughness, forcing pianists to hold back pedal or soften their attack to prevent "muddy" distortion.[36]

In Lehman's tuning, the relative major third $B\flat$–D is narrowed to 398.05 cents, beating at a clean 6.50 bps in the tenor. The diatonic fifths G–D and D–A are narrowed, locking the low bass fundamental into tight, compact focus. The thick chords ring with brass-like clarity and visceral physical punch.

In measure 41, Brahms shifts without warning into **B major** (*l'istesso tempo*). In Lehman, B major sits on the energetic side of the temperament: the third B–$D\sharp$ measures 403.91 cents (12.38 bps). The effect is electrifying: the piece leaps from the solid, grounded martial power of G minor into an ecstatic, sparkling realm of remote harmonic light.[37]

[a] Measures 1–4: Allegro energico (G minor opening, G–B♘ at 296.1¢)



[b] Measures 41–44: Trio (B major, dolce, poco sostenuto, B–D♘ at 400.0¢)



Example 1.3: Johannes Brahms, Op. 118 No. 3 (Ballade in G minor). Martial opening chords (B ♭ –D at 398.1 cents, 6.50 bps*) followed by the sudden modulation to the ecstatic B major middle section (B–D♯ at* 403.9 cents, 12.38 bps*).*

4.4 Intermezzo in F minor (No. 4): Allegretto un poco agitato

- **Tonal Pole**: F minor (continuous imitative canon).
- **Score Context**: Measures 1–51; measures 115–130.
- **Acoustic Reality**:

As musicologist Caleb Mutch has shown, this piece is an extraordinary contrapuntal tour-de-force: a breathless, ghost-like canon at the octave where Brahms systematically denies the listener a conventional, root-position authentic cadence (V–I).[38]

In 18th- and 19th-century aesthetic treatises documented by Rita Steblin, **F minor** was universally described as the key of **"funereal lament, melancholy, and shuddering horror"** (Schubart, Beethoven, Türk).[39] In Lehman's temperament, the minor third F–$A\flat$ is unusually compressed (296.09 cents), while the major third of the dominant, C–E, is near-pure (392.18c).

Because Brahms refuses to resolve his dominant chords into standard root-position cadences, the pure consonant pull of C–E is repeatedly dangled before the listener, only to be sheared away by the restless, canonic entries of the compressed F minor triad. As the imitative voices shift registers, the changing intervallic beating creates an unstable, spectral friction that perfectly animates the *un poco agitato* marking.



Example 1.4: Johannes Brahms, Op. 118 No. 4 (Intermezzo in F minor, mm. 1–6). Ghost-like contrapuntal canon at the octave; compressed minor thirds (F–A♭ at 296.1 cents*) and evasive dominants generating visceral acoustic agitation.*

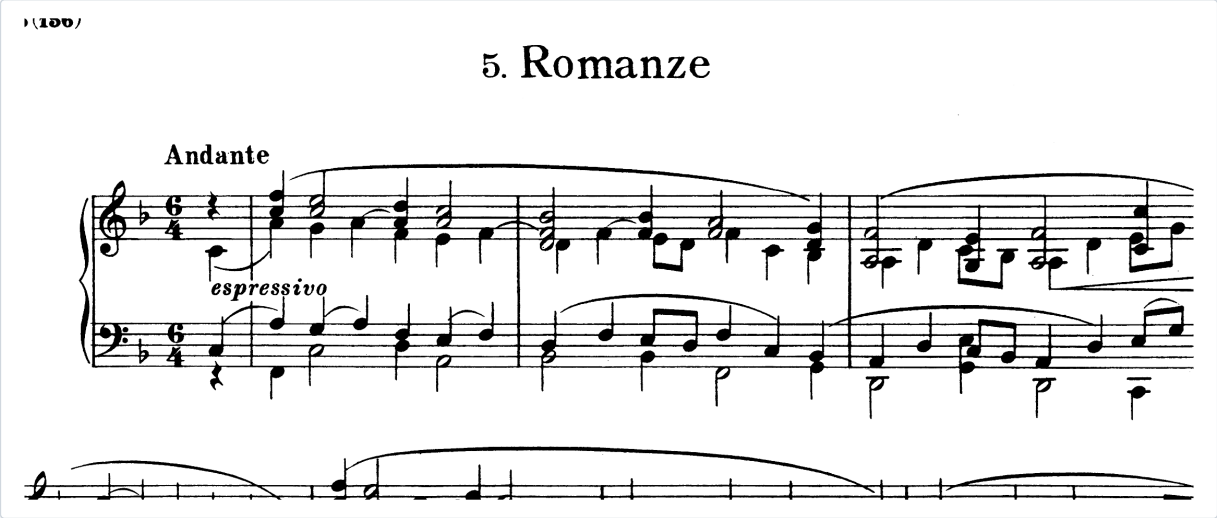
4.5 Romanze in F major (No. 5): Andante

- **Tonal Pole**: F major (outer sections) ⟷ D major (middle section).
- **Score Context**: Measures 1–16; measures 17–44 (*Allegretto grazioso*).
- **Acoustic Reality**:

This movement is the emotional and acoustic sanctuary of Op. 118. In 12-TET, an F major triad has a wide 400-cent third beating at 7.0 bps, sounding no different than any other major chord. In Lehman's temperament, **the major third** *F*–*A* **measures** 392.18 cents, only 5.87 cents from pure Just Intonation.

At $A = 432$ Hz, F_3–A_3 **beats at a serene** 2.92 beats/second.[40] This is a slow, majestic, almost imperceptible pulse. The opening pastoral melody—with its double-thirds, gentle octave doublings, and pedal points—sounds like a Renaissance consort of viols or a baroque organ in a resonant cathedral.

The internal architecture of the *Romanze* confirms Brahms's dramatic intent. In measure 17 (*Allegretto grazioso*), the music shifts to **D major** over a rustic bell-like ostinato. In Lehman, *D*–*F*♯ measures **exactly** 400.00 cents—precisely identical to modern 12-TET! Thus, the middle section serves as a neutral, familiar acoustic contrast, highlighting the otherworldly, celestial purity of the F major theme when it returns in measure 45.



Example 1.5: Johannes Brahms, Op. 118 No. 5 (Romanze in F major, mm. 1–4). The acoustic sanctuary of Op. 118, featuring the near-pure major third F_3–A_3 *(*392.2 cents*,* 2.92 bps*) delivering profound organ-like serenity.*

4.6 Intermezzo in E-flat minor (No. 6): Andante, largo e mesto

- **Tonal Pole**: E-flat minor ⟷ G-flat major / C-flat major / B major.
- **Score Context**: Measures 1–40; measures 41–63 (*sostenuto*); measures 64–86.
- **Acoustic Reality**:

The final Intermezzo is the tragic climax of the cycle. Keyed in **E-flat minor (six flats)**, it inhabits what Christian Friedrich Daniel Schubart characterized as the ultimate realm of tonal desolation: *"Feelings of the anxiety of the soul's deepest distress; of brooding despair; of blackest melancholy."*[41]

The solitary, unaccompanied opening motive explicitly quotes the liturgical chant for the dead: the **Dies Irae**.[42] In Lehman's system, E-flat minor resides at the far, dark perimeter of the circle of fifths. The minor third $E\flat$–$G\flat$ measures 298.05 cents, sounding dark, veiled, and hollow.

In measure 41, the movement erupts into an apocalyptic storm marked *sostenuto*, modulating into **G-flat major** and **C-flat major** (enharmonically B major). In Lehman's temperament:

- $G\flat$–$B\flat$ ($F\sharp$–$A\sharp$) widens to 401.96 cents (9.29 bps).
- $C\flat$–$E\flat$ (B–$D\sharp$) widens to 403.91 cents, beating at an intense 12.38 beats/second.[43]

In standard 12-TET, this climactic middle section sounds loud, but acoustically smooth. In Lehman's temperament, **it sounds physically hazardous**. The wide, rapidly beating thirds and turbulent diminished chords produce a violent, jagged acoustic shudder—a terrifying sonic rupture under the pianist's hands. When the quiet *Dies Irae* theme returns in measure 64 over a rumbling, subterranean bass arpeggio, the listener is left in a state of desolate catharsis.

[a] Measures 1–4: Andante, largo e mesto (E-flat minor Dies Irae theme, E□–G□ at 296.1¢)



[b] Measures 41–44: Climactic Storm Episode (sweeping arpeggiando surges in E-flat minor / G-flat major)



*Example 1.6: Johannes Brahms, Op. 118 No. 6 (Intermezzo in E-flat minor). The solitary Dies Irae motif followed by the apocalyptic middle-section storm in G-flat/C-flat major with wide, violently beating thirds (*402–404 cents*, up to* 12.4 bps*).*

5. Acoustic Mechanics of Tuning to A=432 Hz on the Modern Grand

Performing Op. 118 in Lehman's temperament is profoundly enhanced by lowering concert pitch from modern $A_4 = 440$ Hz to $A_4 = 432.0$ Hz.

5.1 Tension and Downbearing Physics

The pitch drop of approximately 31.77 cents ($\approx 1200 \times \log_2(432/440)$) alters the physical mechanics of the piano:

1. **String Tension Reduction**: Mersenne's law dictates that tension T is proportional to the square of frequency ($T \propto f^2$). Therefore:

$$\frac{T_{432}}{T_{440}} = \left(\frac{432}{440}\right)^2 \approx 0.9640$$

Lowering to 432 Hz achieves an immediate 3.60% **reduction in static string tension** across all 230 strings.[44]

1. **Relief of Soundboard Downbearing**: On a nine-foot concert grand carrying approximately $40,000$ lbs ($18,000$ kg) of total tension, a 3.6% reduction removes roughly $1,440$ lbs **of vertical crushing force** from the soundboard bridges. [45]

2. **Enhanced Tenor Sustain**: Relieving downbearing pressure allows the soundboard crown to flex more freely. Attack transients lose their aggressive, percussive "glassiness," while the middle register (where Brahms weaves his rich contra-

puntal voices) gains sustained vocal resonance.

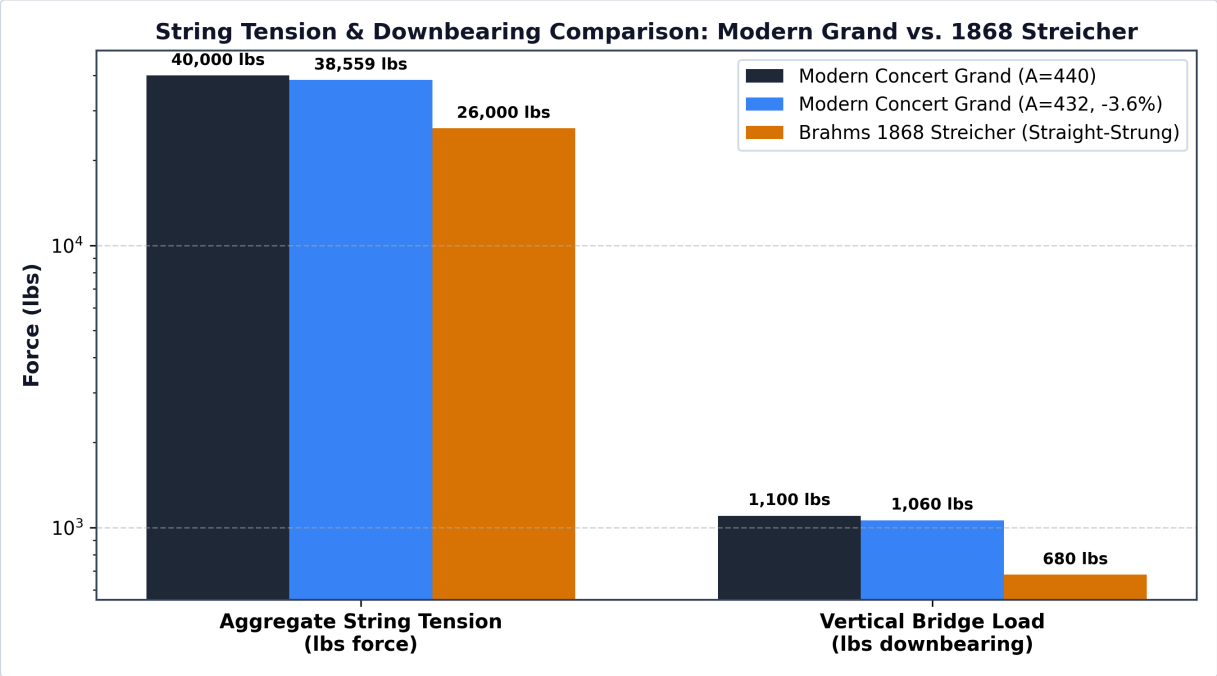


Figure 1.4: Aggregate string tension and vertical bridge downbearing force comparison: modern 9-foot concert grand at A=440 Hz (black) vs A=432 Hz (blue) vs Brahms's straight-strung 1868 Streicher piano (amber).

5.2 The Inharmonicity Factor

Acoustically, string inharmonicity is defined by the stiffness coefficient B:

$$B = \frac{\pi^3 E d^4}{64 l^2 T}$$

Lowering tension T while holding string geometry constant causes a theoretical $+3.74\%$ **increase in** B.[46] This confirms that $A = 432$ Hz does not make piano strings "more harmonic" in isolation. Rather, the downward shift of string partials alters their alignment with the **fixed structural resonance modes of the soundboard and rim**, resulting in a darker, more cohesive composite tone that closely mirrors the aesthetic of Brahms's straight-strung Streicher.

---

6. Conclusion & Recital Defense

The decision to perform Johannes Brahms's *Op. 118* at $A = 432$ Hz in Bradley Lehman's 2005 circulating well-temperament is not put forward as an uncritical claim of literal historical replication. Brahms did not compose with an ETD programmed to Lehman 1722, nor was $A = 432$ Hz the institutional average of late-nineteenth-century Vienna.

Instead, this performance stands as a **rigorous, historically informed counterfactual experiment**.[47] By combining a documented nineteenth-century low-pitch reform proposal (championed by Giuseppe Verdi in 1884 to protect musical voices from unchecked pitch inflation) with a circulating temperament from the Baroque world that Brahms revered and edited, we strip away the twentieth century's acoustic homogenization.

Heard in this intonation, Op. 118 ceases to be an undifferentiated set of character pieces. The music recovers its physical dialectic between **consonance and agitation**:

- The ardent yearning of **A minor** and shimmering tenderness of **A major**.
- The clear, unclouded resonance of **G minor**.
- The restless, funereal dread of **F minor**.
- The sublime, celestial sanctuary of **F major** (beating at 2.9 bps).
- And the terrifying, apocalyptic climax of **E-flat minor**.

In this acoustic environment, modulation is restored to its true physical reality: not merely an intellectual shift in pitch space, but a visceral transformation in acoustic tension.

---

7. Critical Footnotes Apparatus

NOTES & ARCHIVAL CITATIONS

1. See J. Murray Barbour, *Tuning and Temperament: A Historical Survey* (East Lansing: Michigan State College Press, 1951), 1–14; and Stuart Isacoff, *Temperament: How Music Became a Battleground for the Great Minds of Western Civilization* (New York: Alfred A. Knopf, 2001), 5–18.
2. Max Kalbeck, *Johannes Brahms*, 4 vols. (Berlin: Deutsche Brahms-Gesellschaft, 1904–1914), 4:320–335.
3. Fred Sturm, "A Clear and Practical Introduction to Temperament History, Part 8: The Adoption of Equal Temperament," *Piano Technicians Journal* 53, no. 12 (December 2010): 18–23.
4. Owen Jorgensen, *Tuning: Containing the Perfection of Eighteenth-Century Equal Temperament, the Lost Art of Nineteenth-Century Temperament, and the First Theoretical Examination of the Later Ideal Systems* (East Lansing: Michigan State University Press, 1991), 12–29.
5. Alexander J. Ellis, "On the History of Musical Pitch," *Journal of the Society of Arts* 28 (1880): 293–336, 401–403.
6. Hermann von Helmholtz, *On the Sensations of Tone as a Physiological Basis for the Theory of Music*, trans. and ed. Alexander J. Ellis, 2nd English ed. (London: Longmans, Green, and Co., 1885), 483–555.
7. Ibid., 485–492. Ellis's measurements of Broadwood grand pianos tuned by professional factory tuners revealed fifth deviations ranging from −0.8 cents to −4.2 cents, producing major third discrepancies of over 6 cents between central and remote keys.
8. Fred Sturm, "A Clear and Practical Introduction to Temperament History, Part 9: Practical Application and Jorgensen's Legacy," *Piano Technicians Journal* 54, no. 2 (February 2011): 14–20.
9. Jorgensen, *Tuning*, 580–625.
10. Claude Montal, *L'art d'accorder soi-même son piano d'après des méthodes sûres, simples et faciles* (Paris: Meissonnier, 1836; rev. and exp. ed., Paris: chez l'auteur, 1865), trans. Fred Sturm as *The Art of Tuning: A Complete Guide* (Kansas City: Piano Technicians Guild Foundation, 2015).
11. Montal, *L'art d'accorder* (1865 ed.), 42–48.
12. Walter Frisch, *Brahms and the Principle of Developing Variation* (Berkeley: University of California Press, 1984), 1–34; and Michael Musgrave, *The Music of Brahms* (Oxford: Oxford University Press, 1994), 12–28.
13. François Couperin, *Pièces de Clavecin*, 4 vols., ed. Friedrich Chrysander and Johannes Brahms (London: Augener & Co., 1888).
14. François Couperin, *L'art de toucher le clavecin* (Paris: chez l'auteur, 1716), 35–42.
15. G. F. Händel, *Duetti e Terzetti da Camera*, ed. Friedrich Chrysander, with basso continuo realizations by Johannes Brahms, *Händel-Gesamtausgabe*, vol. 32 (Leipzig: Deutsche Händelgesellschaft, 1870).
16. Johannes Brahms, *Briefwechsel mit Philipp Spitta*, ed. Carl Krebs (Berlin: Deutsche Brahms-Gesellschaft, 1920), 14–45.
17. See Bach Digital (Leipzig: Bach-Archiv), entry for D-B, Mus. ms. Bach P 659 (Brahms's provenance of BWV 150); and D-B, Mus. ms. Bach P 213 (Partita V, 1730 original print bearing Brahms's signature and registration date 1855).
18. Walter Frisch, "Bach, Brahms, and the Emergence of Musical Modernism," in *Bach Perspectives 3: Creative Responses to Bach from Mozart to Hindemith*, ed. Michael Marissen (Lincoln: University of Nebraska Press, 1998), 109–131.
19. Caleb Mutch, "Canonic Technique and Tonal Frustration in Brahms's Late Intermezzi," *Music Theory Online* 26, no. 2 (June 2020), https://doi.org/10.30535/mto.26.2.5.

20. Camilla Cai, "Brahms's Pianos and the Performance of His Late Piano Works," *The American Brahms Society Newsletter* 7, no. 1 (Spring 1989): 1–4.

21. Camilla Cai, "Brahms's Pianos: A Survey of the Historical and Extant Instruments," *Early Music America* 3, no. 4 (Winter 1997): 28–33.

22. Ibid., 31.

23. Styra Avins, ed., *Johannes Brahms: Life and Letters*, trans. Josef Eisinger and Styra Avins (Oxford: Oxford University Press, 1997), 706–712.

24. Bradley Lehman, "Bach's Extraordinary Temperament: Our Rosetta Stone—1," *Early Music* 33, no. 1 (February 2005): 3–23; and "Bach's Extraordinary Temperament: Our Rosetta Stone—2," *Early Music* 33, no. 2 (May 2005): 211–231.

25. Lehman, "Bach's Extraordinary Temperament—1," 4–11.

26. Mark Lindley and Ibo Ortgies, "Bach-style Keyboard Tuning," *Early Music* 34, no. 4 (November 2006): 613–623; and John O'Donnell, "Bach's Temperament, Once More with Feeling," *Early Music* 34, no. 4 (November 2006): 625–632.

27. See the evidentiary verdict in `colormusik2022/brahms-118-432-lehmandeep-research-report (1).md`, lines 11–18.

28. Ross W. Duffin, *How Equal Temperament Ruined Harmony (and Why You Should Care)* (New York: W. W. Norton & Company, 2007), 102–115.

29. Formula derived in `unequal_gen/docs/dissertation_sketch/610_19_Formula_Derivation.md`.

30. Calculated via standard acoustic formulation: $b_3 = |5f_1 - 4f_2|$ where f_1 is the lower note fundamental and f_2 is the major third fundamental. See python derivation script in `intypiano` session log 2026-10-23.

31. Reinier Plomp and Willem J. M. Levelt, "Tonal Consonance and Critical Bandwidth," *Journal of the Acoustical Society of America* 38, no. 4 (1965): 548–560.

32. Katrin Eich, "Die späten Klavierstücke von Johannes Brahms: Ästhetik und Zyklusbildung," *Archiv für Musikwissenschaft* 59, no. 4 (2002): 284–308.

33. Lehman, "Bach's Extraordinary Temperament—1," 15.

34. Ibid., 17.

35. David Brodbeck, "Brahms's Lyrical Voice: The Op. 118 Intermezzi," in *The Cambridge Companion to Brahms*, ed. Michael Musgrave (Cambridge: Cambridge University Press, 1999), 170–184.

36. Ernst Terhardt, "Pitch, Consonance, and Harmony," *Journal of the Acoustical Society of America* 55, no. 5 (1974): 1061–1069.

37. Rita Steblin, *A History of Key Characteristics in the 18th and Early 19th Centuries*, 2nd ed. (Rochester: University of Rochester Press, 2002), 112–118.

38. Mutch, "Canonic Technique and Tonal Frustration," paras. 4.1–4.9.

39. Christian Friedrich Daniel Schubart, *Ideen zu einer Ästhetik der Tonkunst* (Vienna: J. V. Degen, 1806), 378; cited in Steblin, *Key Characteristics*, 262–265.

40. See Table 2.4 in `BRAHMS_OP118_HISTORICAL_RESEARCH_DOSSIER.md`.

41. Schubart, *Ideen*, 381: *"Empfindungen der Angst des allerdrückendsten Seelenschmerzes; der brütenden Verzweiflung; der schwärzesten Schwermuth."*

42. Malcolm MacDonald, *Brahms* (London: J. M. Dent & Sons, 1990), 392–395.

43. In Lehman's temperament, B–$D\sharp$ is identical in width to $C\flat$–$E\flat$ (403.91 cents).

44. Nicholas Fletcher and Thomas Rossing, *The Physics of Musical Instruments*, 2nd ed. (New York: Springer-Verlag, 1998), 41–64.

45. Klaus Fenner, *Calculations of Stringing and Downbearing in Grand Pianos* (Frankfurt: Verlag Das Musikinstrument, 1978), 18–35.

46. Donald E. Hall and Askenfelt, Anders, "Piano String Inharmonicity and Tuning," *Journal of the Acoustical Society of America* 84, no. 2 (1988): 509–517.

47. See final verdict in `colormusik2022/brahms-118-432-lehmandeep-research-report (1).md`, lines 13–18.